INVESTIGATION OF THE SOLIDIFICATION OF A JET OF VISCOUS LIQUID, NOT MIXING WITH THE SURROUNDING MEDIUM

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With the propagation of a jet of viscous liquid, not mixing with the surrounding medium, having a temperature lower than the solidification temperature of the outflowing liquid, there is solidification of the jet starting from some part of the surface. If it is assumed that the process is equilibrium, solidification of the jet takes place at the point where the liquid at the surface of the jet attains the solidification temperature T_S. With a steady-state process, the region of solidification is propagated downstream, gradually expanding and taking in all the new masses of liquid in the jet. Similar processes take place with the formation, for example, of synthetic fibers and films from polymer melts [1].

Let us consider the problem of the simplification of a jet for the simplest model of a liquid, i.e., a Newtonian model. In addition, the model of a Newtonian liquid can serve as a good approximation for some more complex liquids.

At a physical model of the process we use the scheme shown in Fig. 1. At some point x_0 let the temperature of the liquid at the surface attain the solidification temperature T_S. From this point, region 3 starts to propagate, i.e., a region of solidifying liquid. The boundary between the liquid in the jet and the solid material is the surface of a phase transition. Region 1 is the external boundary layer; region 2 is a new boundary layer, formed as the result of a sharp change in the boundary condition at the surface of the jet at the point x_0. Communications [2, 3] are devoted to the development of such a boundary layer and to the determination of the flow parameters in it. Region 4 is occupied by a liquid moving in a jet.

Let us construct the solution in the neighborhood of the point x_0, using the method of [2, 3], based on the method of joining asymptotic expansions [4]. As a small parameter we use the value of $\varepsilon = \delta_2^*/\delta^*$, i.e., the ratio of the thickness of the displacement of the new boundary layer to the thickness of the displacement of the whole boundary layer. In what follows we shall assume that the thickness of a filament is equal to a; the flow in region 4 is isobaric (the pressure of the liquid is equal to the external pressure p_a); the transverse gradients are considerably greater than the longitudinal, as a result of which the theory of the boundary layer can be used. We write the principal equations (the liquid is assumed to be ponderable and incompressible; the gas is imponderable and incompressible; the physical parameters of the medium are constant) in the form

$$u_i\frac{\partial u_i}{\partial x}+v_i\frac{\partial u_i}{\partial y}=\delta_{i4}g+\nu_i\frac{\partial^2 u_i}{\partial y^2},\quad \delta_{i4}=\begin{cases}1, & i=4,\\ 0, & i\neq 4,\end{cases}$$

$$\frac{\partial u_i}{\partial x}+\frac{\partial v_i}{\partial y}=0\quad (i=1,2,4),$$

$$\rho_i c_i\left(u_i\frac{\partial T_i}{\partial x}+v_i\frac{\partial T_i}{\partial y}\right)=\lambda_i\frac{\partial^2 T_i}{\partial y^2}\quad (i=1,2,3,4).$$

For region 3, $u_3 = U_S = \text{const}$; $v_3 = 0$. In what follows we assume that $\{\rho_{1,2}, c_{1,2}, \nu_{1,2}, \lambda_{1,2}\} = \{\rho_g, c_g, \nu_g, \lambda_g\}$; $\{\rho_2, c_2, \lambda_2\} = \{\rho_S, c_S, \lambda_S\}$; $\{\rho_4, c_4, \nu_4, \lambda_4\}$; $\{\rho_L, c_L, \nu_L, \lambda_L\}$.

In these formulas, u is the velocity in the direction of the x axis; v is the velocity in the direction of the y axis; T is the temperature; $g = 9.8\ \text{m/sec}^2$; ρ is the density; c is the heat capacity; λ is the thermal conductivity; ν is the kinematic viscosity.

1. Region 1. Introducing the stream function ($u_i = \partial\psi_i/\partial y$, $v_i = -\partial\psi_i/\partial x$), we write the solution in region 1 in the following form:

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$$\psi_1 = U\delta^* [F_0(h) + \varepsilon F_1(h;\Delta) + \varepsilon^2 F_2(h;\Delta) + \dots],$$
$$Q_1 = \delta^* T_S [q_0(h) + \varepsilon q_1(h;\Delta) + \varepsilon^2 q_2(h;\Delta) + \dots], \tag{1.1}$$
$$h = \frac{y}{\delta^*}, \quad \Delta = \frac{\delta^*}{\delta_0^*},$$

where $Q = \int (T - T_\infty) dy$; $\delta^* = \int_0^\infty (u - u_\infty)\, dy$; δ_0^* is the thickness of the displacement at the point x_0. The functions F_0 and q_0 determine the profiles of the velocities and the temperatures in the external boundary layer in the cross section x_0, and are assumed to be known. Before substituting (1.1) into the main equations, we must set

$$\frac{d\varepsilon}{dx} = \frac{1}{2}\frac{\nu g}{U\delta^{*2}}\varepsilon^{-1}(1 + \varepsilon\gamma_1 + \varepsilon^2\gamma_2 + \dots), \tag{1.2}$$
$$\frac{d\delta^*}{dx} = \frac{1}{2}\frac{\nu g}{U\delta^*}\varepsilon^{-1}(\alpha_0 + \varepsilon\alpha_1 + \varepsilon^2\alpha_2 + \dots),$$

where γ_j and α_j are some values. After substitution of the solutions, we obtain equations for determining F_j and q_j, admitting of direct integration, as a result of which we obtain

$$F_1 = -\alpha_0(F_0 - hF_0') + k_1 F_0',$$
$$F_2 = -\frac{1}{2}(\alpha_1 - \alpha_0^2)(F_0 - hF_0') + \frac{1}{2}\alpha_0^2 h^2 F_0'' - \frac{1}{2}\gamma_1 F_1 + k_2 F_0' + \frac{1}{2}k_1^2 F_0'' + k_1\alpha_0 h F_0'' + F_0'\int_0^h \frac{F_0'''}{F_0'^2}dh;$$
$$q_1' = (k_1 + \alpha_0 h)\, q_0'',$$
$$q_2' = k_2 q_0'' + \frac{1}{2}k_1^2 q_0''' + \frac{1}{2}\alpha_0^2 (h^2 q_0'')' + k_1\alpha_0 (h q_0'')' - \frac{1}{2}\gamma_1(k_1 q_0'' + \alpha_0 h q_0'') + \frac{1}{2}(\alpha_1 - \alpha_0^2) h q_0'' + q_0''\int_0^h \frac{F_0'''}{F_0'^2}dh + \mathrm{Pr}_g^{-1}\frac{q_0'''}{F_0'^2}.$$

To determine the term $j = F_0'\int_0^h \frac{F_0'''}{F_0'^2}dh$ a concrete expression for F_0 is needed. For the case of a jet issuing downward, the asymptotic solution of [5] can be used. If it is taken into consideration that ψ and Q, far from the source, up to the solidification point can be represented as $\psi = x^l F_0\left(\frac{y}{x^k}\right)$, $Q = x^p q_0\left(\frac{y}{x^k}\right)$, then, from the boundary-layer equations it follows that $F_0''' = (l-k)F_0'^2 - lF_0F_0''$, $q_0''' = (p-k)F_0'q_0' - lF_0q_0''$.

Using the expression for F_0''', we can obtain $j = -khF_0' + lF_0'$; then, from the expressions at infinity it follows that (the velocity of the external flow is equal to zero) $\alpha_0 = 0$, $\alpha_1 = l$. Now, using the expression for q_0''' it can be shown that, with $h \to \infty$, q_1' and q_2' tend toward zero.

Since the value of ε is of a somewhat arbitrary character, there is a certain degree of arbitrariness in the choice of the coefficients γ_j. From Eqs. (1.2) we can obtain

$$\delta^* = \left(\frac{\nu g x}{U}\right)^{1/2}(\beta_0 + \beta_2\varepsilon^2 - \beta_3\varepsilon^3 + \dots),$$
$$\beta_2 = \frac{1}{2}\alpha_1 - \frac{1}{4}\beta_0, \quad \beta_3 = \frac{\alpha_2}{3} - \frac{2}{3}\gamma_1\beta_2,$$

β_0 is a known quantity. Taking into account that, with $x \to \infty$, $\varepsilon \to 1$, and $d\varepsilon/dx \to 0$, we set

$$1 + \varepsilon\gamma_1 + \varepsilon^2\gamma_2 + \dots = (1 - \varepsilon^2)\left(1 + \varepsilon^2\frac{\beta_2}{\beta_0} + \varepsilon^3\frac{\beta_3}{\beta_0} + \dots\right),$$

then $\gamma_1 = 0$, $\gamma_2 = -(1 - \beta_2/\beta_0)$,

$$\varepsilon = \sqrt{1 - \left(\frac{x_0}{x}\right)^{1/\beta_0^2}}.$$

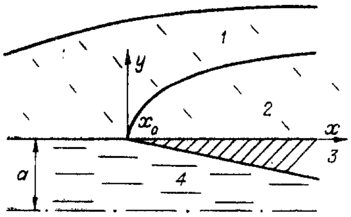

Fig. 1

The coefficients k_j can be determined from the conditions for the joining of the solutions (1.1) and the solutions in region 2.

2. Region 2. We represent the solutions in the new boundary layer in the form

$$\psi_2 = \varepsilon\delta^* U[G_0(H;\ \Delta) + \varepsilon G_1(H;\ \Delta) + \ldots],$$
$$Q_2 = \varepsilon\delta^* T_S[Q_0(H;\ \Delta) + \varepsilon Q_1(H;\ \Delta) + \ldots],\quad H = y/\varepsilon\delta^*.$$

From the equations of the boundary layer, we can obtain equations for G_j

$$G_0''' + \frac{1}{2} G_0 G_0'' = 0,$$
$$G_1''' - \frac{1}{2}\left(G_0' G_1' - 2G_0'' G_1 - G_0 G_1''\right) = -\frac{\gamma_1}{2} G_0 G_0'',$$
$$G_2''' - \frac{1}{2}\left(2G_0' G_2' - 3G_0'' G_2 - G_0 G_2''\right) = \frac{\gamma_2}{2}\left(G_1'^2 - 2G_1 G_1''\right) + \frac{\gamma_1}{2}\left(G_0' G_1' - 2G_0'' G_1 - G_0 G_1''\right) - \frac{\gamma_2 + \alpha_1}{2} G_0 G_0'';$$

for Q_j

$$Q_0''' + \frac{1}{2}\Pr_g G_0 Q_0'' = 0,$$
$$Q_1''' - \frac{1}{2}\Pr_g\left(G_0' Q_1' - G_0 Q_1''\right) = -\frac{1}{2}\Pr_g\left(G_1 Q_0'' + \gamma_1 G_0 Q_0''\right),$$
$$Q_2''' - \frac{1}{2}\Pr_g\left(2G_0' Q_2' - G_0 Q_2''\right) = \frac{1}{2}\Pr_g\left[G_1' Q_1' - 2G_1 Q_1'' - 3G_2 Q_0'' + \right.$$
$$\left. + \gamma_1\left(G_0' Q_1' - G_1 Q_0''\right) - \gamma_1\left(G_1 Q_0'' + G_0 Q_1''\right) - (\gamma_2 + \alpha_1) G_0 Q_0''\right].$$

The boundary conditions at a moving filament for the flow function can be written in the form

$$G_j(0;\Delta) = 0,\quad G_0'(0;\Delta) = \frac{U_S}{U},\quad G_j'(0;\Delta) = 0\quad (j > 0).$$

The velocity of a filament is determined from the condition of the conservation of mass

$$U_S = \frac{\rho_L}{\rho_S}\int_0^1 U f_0'(n)\, dn$$

($Uf_0'(n)$ is the profile of the velocities in the cross section x_0 in the jet); in the general case $U_S \neq U$. Let us consider the case where $U_S = U$, which holds out the possibility of obtaining simple asymptotic expressions for G_j

$$G_0 = H,\ G_1 = B_1 H^2,\ G_2 = B_2(H^3 + 6H)$$

and for Q_j

$$Q_0' = M_0,\quad Q_1' = K_1\left(\chi + \frac{1}{2}\Pr_g H\int_0^H \chi dH\right) + M_1 H,$$

$$Q_2' = -\frac{1}{2} B_1 K_1 H\chi + K_2\left(\int_0^H \chi dH + H\chi + \frac{1}{2}\Pr_g H^2\int_0^H \chi dH\right) + M_2\left(H^2 + 2\Pr^{-1}\right),\quad \chi = e^{-\frac{1}{4}\Pr_g H^2} \qquad (2.1)$$

Here, for brevity, certain terms which are equal to zero are not written out. Using the rule for the joining of asymptotic solutions, we obtain

$$B_1 = \frac{1}{2} F_0''(0), \quad B_2 = \frac{1}{3!} F_0'''(0); \tag{2.2}$$

$$k_1 = 0, \quad k_2 = 0; \quad M_1 = 1 - \frac{T_\infty}{T}, \quad \frac{1}{2} K_1 \mathrm{Pr_g} \int_0^\infty \chi dH + M_1 = q_0''(0),$$

$$\frac{1}{2} K_2 \mathrm{Pr_g} \int_0^\infty \chi dH + M_2 = \frac{1}{2} q_0'''(0).$$

The values of $K_j(\Delta)$ and $M_g(\Delta)$ can be found from the joining of the solutions at the boundaries of regions 2-3 and 3-4.

3. Region 3. In region 3 we set

$$Q_s = \varepsilon^2 \delta^* T_s (\beta_2 + \varepsilon\beta_3 + \ldots)[g_0(N;\ \Delta) + \varepsilon g_1(N;\ \Delta) + \ldots],$$

$$N = \frac{y}{\varepsilon^2 \delta^* (\beta_2 + \varepsilon\beta_3 + \ldots)},$$

where $\beta_j(\Delta)$ are some values, determining the position of the phase interface. The equation of this surface is determined by the expression $N = -1$, i.e., $y_* = -\varepsilon^2\delta^*(\beta_2 + \varepsilon\beta_3 + \ldots)$. For the functions g_j' we have the simple dependences

$$g_0' = C_{00} + C_{01}N, \quad g_1' = C_{10} + C_{11}N, \quad g_2' = C_{20} + C_{21}N, \quad g_3' = C_{30} + C_{31}N, \tag{3.1}$$

where $C_{ij}(\Delta)$ can be determined from the boundary conditions with $N = 0$ and $N = -1$.

4. Region 4. In this region, we write the solutions in the form of the following expansions (the coordinate y is reckoned from the axis of the jet):

$$\begin{aligned} \psi_4 &= Ub[f_0(n) + \varepsilon f_1(n;\ \Delta) + \varepsilon^2 f_2(n;\ \Delta) + \ldots], \\ Q_4 &= T_s b[\theta_0(n) + \varepsilon\theta_1(n;\ \Delta) + \varepsilon^2\theta_2(n;\ \Delta) + \ldots], \\ b &= a - \varepsilon^2\delta^*(\beta_2 + \varepsilon\beta_3 + \ldots), \quad n = y/b. \end{aligned} \tag{4.1}$$

After substitution of (4.1) into the equations of the boundary layer, we obtain the expressions

$$f_1 = 0, \quad f_2 = \beta_2 \frac{\delta_0^*}{a} \Delta (f_0 - nf_0') + \frac{\delta_0^{*2}}{a^2} \Delta^2 \frac{\mathrm{Re_g}}{\mathrm{Fr}} f_0' \int_0^n \frac{dn}{f_0'^2} + \frac{\delta_0^{*2}}{a^2} \Delta^2 \frac{\nu_L}{\nu_g} f_0' \int \frac{f_0'''}{f_0'^2} dn, \tag{4.2}$$

$$f_3 = -\frac{2}{3} \gamma_1 f_2 + \frac{\delta_0^*}{a} \Delta \left(\beta_3 + \frac{2}{3} \beta_2 \gamma_1\right)(f_0 - nf_0');$$

$$\theta_1 = 0, \tag{4.3}$$

$$\theta_2 = -\beta_2 \frac{\delta_0^*}{a} \Delta n \theta_0' + \frac{\delta_0^{*2}}{a^2} \Delta^2 \frac{\mathrm{Re_g}}{\mathrm{Fr}} \theta_0' \int_0^n \frac{dn}{f_0'^2} + \frac{\delta_0^{*2}}{a^2} \Delta^2 \frac{\nu_L}{\nu_g} \theta_0' \int_0^n \frac{f_0'''}{f_0'^2} dn + \frac{\delta_0^{*2}}{a^2} \Delta^2 \mathrm{Pr_L^{-1}} \frac{\nu_L}{\nu_g} \frac{\theta_0''}{f_0'}.$$

In finding these solutions it was assumed that, as a result of symmetry

$$f_j(0) = 0, \quad f_j''(0) = 0, \quad \theta_j''(0) = 0.$$

The functions $f_0(n)$ and $\theta_0(n)$ are assumed to be known. They determine the profiles of the velocities and temperatures in the jet in the cross section x_0.

5. Determination of Phase Interface. Since the flow in region 4 is assumed to be isobaric, then, at the liquid–solid phase interface the condition is satisfied

$$T = T_s. \tag{5.1}$$

Another condition must be the equality of the heat fluxes, taking account of the latent heat of the phase transition. In the statement adopted, the boundary layer in a liquid adjacent to a solid phase is excluded from the discussion, i.e., here a model of a flow with a phase transition, with slip of the phases at the interface and a jump in the temperatures, is adopted. Such a model is frequently used with the flow of condensate films [6]. In this case, the boundary condition at the interface has the form

$$-\lambda_S \frac{\partial T_S}{\partial y}\Big|_{y*} = r\rho_L v_4|_{y*} + \rho_L v_4|_{y*} c_L (T_4 - T_S)|_{y*}, \tag{5.2}$$

where r is the heat of the phase transition.

At the surface of a moving filament, the equalities of the temperatures and the heat fluxes must be satisfied, i.e.

$$T_2|_{y=0} = T_S|_{y=0}; \tag{5.3}$$

$$\lambda_g \frac{\partial T_2}{\partial y}\Big|_{y=0} = \lambda_S \frac{\partial T_S}{\partial y}\Big|_{y=0}. \tag{5.4}$$

Using the analytical dependences (2.1), (3.1), (4.2), and (4.3), from the boundary conditions (5.1)-(5.4) and the relationships (2.2) we find

$$C_{00} = 1 - \frac{T_\infty}{T_S}, \quad C_{01} = C_{10} = C_{11} = 0, \quad C_{20} = C_{21} = -2\frac{\rho_L v_g}{\lambda_S}\frac{r}{T_S}\beta_2 \times$$
$$\times \left[\beta_2 - \frac{\delta_0^*}{a}\Delta\left(\frac{\mathrm{Reg}}{\mathrm{Fr}}\int_0^1 \frac{dn}{f_0'^2} + \frac{v_L}{v_g}\int_0^1 \frac{f_0'''}{f_0'^2}dn\right)\right],$$
$$C_{30} = C_{31} = -\frac{\rho_L v_g}{\lambda_S}\frac{r}{T_S}\left[5\beta_2\beta_3 - 2\frac{\delta_0^*}{a}\Delta\beta_3\left(\frac{\mathrm{Reg}}{\mathrm{Fr}}\int_0^1 \frac{dn}{f_0'^2} + \frac{v_L}{v_g}\int_0^1 \frac{f_0'''}{f_0'^2}dn\right)\right],$$
$$K_1 = 0,$$
$$M_1 = q_0''(0), \quad K_2 = -\frac{3}{2}\frac{\rho_L v_g}{\lambda_g}\frac{r}{T_S}\beta_3, \quad M_2 = \frac{\mathrm{Prg}}{2}C_{20}.$$

Finally, from the boundary conditions (5.4) we determine

$$\beta_2 = \frac{\delta_0^*}{a}\Delta\left(\frac{\mathrm{Reg}}{\mathrm{Fr}}\int_0^1 \frac{dn}{f_0'^2} + \frac{v_L}{v_g}\int_0^1 \frac{f_0'''}{f_0'^2}dn\right) - \frac{1}{2}\frac{\lambda_g}{\rho_L v_g}\frac{T_S}{r}q_0''(0) \quad (q_0''(0) < 0),$$
$$\beta_3 = -\frac{4}{3}\frac{\lambda_g}{\mathrm{Pr}_g \rho_L v_g}\frac{T_S}{r}\left(\int_0^\infty \chi dH\right)^{-1}\left\{\frac{1}{2}q_0'''(0) - \frac{1}{2}\mathrm{Pr}_g\frac{\lambda_g}{\lambda_S}\beta_2 q_0''(0)\right\}. \tag{5.5}$$

Thus, analytical expressions have been obtained for the flow parameters in the neighborhood of the point x_0. Within the framework of boundary-layer theory, the point x_0 is a singular point.

With $U \neq U_S$, in the neighborhood of this point, τ_W rises unboundedly; in the case under consideration, the continuity both of the velocities and the temperatures, as well as of the friction stress and the heat flux from the jet to the gas, is conserved. Let us write formulas for the temperature and the heat flux at the surface of a filament

$$T = T_S\left\{1 + \varepsilon^2 \frac{\lambda_g}{\lambda_S}\beta_2 q_0''(0)\right\},$$

$$q = -\lambda_g T_S \delta^{*-1}\left[q_0''(0) - \varepsilon\frac{3}{2}\frac{\rho_L v_g}{\lambda_g}\frac{r}{T_S}\beta_3\right],$$

from which it can be seen that the continuity of T and q at the point x_0 is retained.

An interesting special characteristic of the solidification process under consideration is the fact that the region 3 appears in the form of a sharp edge. This can be seen from the expression $y_* = -\varepsilon^2\delta^* (\beta_2 + \varepsilon\beta_3 + \ldots)$, differentiating which with respect to x we will have $\left.\frac{dy_*}{dx}\right|_{x_0} = -\frac{\nu_g}{U\delta_0^*}\beta_2$, i.e., the angle between a tangent to the surface and the x axis has a finite value.

It must also be added that the formulas obtained (5.5) ($U = U_S$) are valid also in the case where there is a velocity of the external flow. Under these circumstances, the effect of the value and direction of the velocity will appear through the value of $q_0''(0)$. If we set $Pr_g = 1$, from the Crocco integral [7], we can obtain

$$q_0' = \frac{u - U_\infty}{U - U_\infty}\left(1 - \frac{T_\infty}{T_S}\right), \text{ then } q_0''(0) = \mu_g^{-1}\frac{\tau_w}{U - U_\infty}\left(1 - \frac{T_\infty}{T_S}\right).$$

This value can serve for an evaluation of the effect of the velocity of the flow and the temperature of the external medium on the solidification process.

LITERATURE CITED

1. A. T. Serkov (editor), Theory of the Formation of Chemical Fibers [in Russian], Khimiya, Moscow (1975).
2. V. I. Eliseev, "Theory of a boundary layer with suddenly changing boundary layers," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1972).
3. V. I. Eliseev, "The construction of solutions of the equations of a compressible boundary layer at a plate with suddenly changing boundary conditions," Inzh.-Fiz. Zh., 24, No. 3 (1973).
4. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Parabolic Press (1975).
5. V. I. Eliseev, "Asymptotic solution of problems of the outflow of heavy laminar jets of incompressible liquids," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1977).
6. V. I. Isachenko, Heat Transfer with Condensation [in Russian], Izd. Énergiya, Moscow (1977).
7. H. Schlichting and K. Schlichting, BoundaryLayer Theory, McGraw-Hill (1968).

TRANSONIC FLOWS OF GAS IN LAVAL NOZZLES WITH LOGARITHMIC SPECIAL FEATURES IN THEIR LIMITING CHARACTERISTICS

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In flat Laval nozzles there are three types of asymptotic flows in the neighborhood of the center [1]. This conclusion was reached using the theorem of Brio and Buke [2] with respect to the behavior, near a singular point (image of the limiting characteristic), of the general integral of the ordinary differential equation to which a study of self-similar transonic flows reduced. It has been found that, with some values of the index of the self-similarity, any given integral curve can be analytically prolonged through this singular point, which is a mesh point in the problem of nozzle flows. With the consideration of flows in nozzles with a round transverse cross section, it is useful to consider the same indices which are considered in the theorem. It is well known [3] that, in this case, there is a second possible alternative: None of the integral curves passing through the mesh point, with the exception of an isolated whisker, yield an analytical continuation. This relates also to a whisker of general direction. In other words, a whisker of general direction holds out the possibility of an analytical prolongation with any given self-similarity index, with the exception of those considered in the theorem. In [4], a second asymptotic type of flow in the neighborhood of the center of an axisymmetric nozzle is constructed numerically

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